K_2^0 Decay*

D. LUERS,† I. S. MITTRA,‡ W. J. WILLIS, AND S. S. YAMAMOTO
Brookhaven National Laboratory, Upton, New York
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A sample of 479 K_2^0 decays in the BNL 20-in. hydrogen bubble chamber was obtained. Of these, 153 are identified as $K_2^0 \to (\pi^\pm e^\mp \nu)$ decay. The validity of CP invariance is suggested by the upper limit of 1.5% we are able to place on the fraction of all K_2^0 decays which result in two charged pions; and by the ratio $(\pi^+e^-\nu)/(\pi^-e^+\nu)=0.99\pm0.16$. The $(\pi^\pm e^\mp \nu)$ decay is found to proceed by the vector interaction, with an approximately constant form factor. The ratio $K_{\mu 3}{}^0/K_{e3}{}^0=0.73\pm0.15$ implies that the ratio of the two form factors appearing in $K_{\mu 3}$ decay f_-/f_+ is consistent with zero. The fraction of visible K_2^0 decays which result in $(\pi^+\pi^-\pi^0)$ is 0.157 ± 0.03. The prediction obtained by using the $\Delta I=\frac{1}{2}$ currents is 0.152 ± 0.017. Our data may be combined with other experiments to provide additional tests of the $\Delta I=\frac{1}{2}$ rule.

I. INTRODUCTION

THE prediction by Pais and Gell-Mann[1] of the existence of the K_2^0 meson was verified by the experiment of Bardon, Lande, Ledermann, and Chinowsky.[2] In that experiment it was demonstrated that a long-lived neutral meson existed, decaying into three or more particles, with approximately the mass of the well-known K_1^0 meson which decays into two pions. Some information on the decay modes was obtained, as well as a measurement of the lifetime. Since that time several of the phenomena peculiar to the $K_1^0-K_2^0$ complex have been observed, such as the mixed positive and negative strangeness of the K_2^0,[3] and the coherent regeneration of K_1^0 mesons from a K_2^0 beam traversing an absorber.[4]

The K_2^0 decay modes for which there is experimental evidence are[1,5]:

$$K_2^0 \to \pi^\pm + e^\mp + \nu, \quad \text{or} \quad (\pi e \nu); \tag{1}$$

$$K_2^0 \to \pi^\pm + \mu^\mp + \nu, \quad \text{or} \quad (\pi \mu \nu); \tag{2}$$

$$K_2^0 \to \pi^\pm + \pi^\mp + \pi^0, \quad \text{or} \quad (\pi^+\pi^-\pi^0); \tag{3}$$

$$K_2^0 \to \pi^0 + \pi^0 + \pi^0, \quad \text{or} \quad (\pi^0\pi^0\pi^0). \tag{4}$$

In this paper it will be assumed that other decay modes can be neglected. This is in accord with the results on K^+ decay.[6]

Many experiments on these decays have been proposed. A question basic to the concept of the K_2^0 is that of the conservation of CP (charge conjugation times parity). Since the K_2^0 is an eigenvalue of CP equal to minus one, it should not decay into two pions. By placing an upper limit on such decays a limit is placed on the degree of violation of CP invariance, subject to a reservation which will be discussed later. A consequence of CP conservation applied to the decay (1) is that the rates of $(\pi^+e^-\nu)$ and $(\pi^-e^+\nu)$ should be equal, which can be tested for events in which the particle masses are identified.[7]

If the weak interactions are in some sense "universal," K mesons should decay by the V-A interaction known from the decay of nonstrange particles,[8] but prior to this experiment this fact had not been established. For decays (1) and (2) this implies the interaction should be vector, rather than scalar or tensor.

The character of the decay interaction can be determined by studying the correlations among the particles resulting from the decay,[9] just as the interaction in nuclear β decay has been studied by means of angular correlation experiments. This determination is particularly straightforward for decay (1), $(\pi e \nu)$, where only one form factor is involved. An advantage of the K_2^0 decay is that events are kinematically determined by measurements on the charged decay products, while gamma rays must be measured to perform the corresponding experiments with K^+.

Decay (2), $(\pi\mu\nu)$ involves two form factors, one of which makes a negligible contribution to $(\pi e \nu)$ decay since it is proportional to the mass of the lepton. The ratio of these form factors can be measured by determining the relative rates of decays (1) and (2).[9]

The possibility of the measurement of decay rates from $K^\pm$, K^0, K_1^0, and K_2^0 has led to study of the rela-

* Research carried out under the auspices of the U. S. Atomic Energy Commission.
† Present address: Max-Planck Institut für Physik, Munich, Germany.
‡ Present address: Panjab University, Chandigarh, Panjab, India.

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tions among these rates predicted by assumptions concerning the charge dependence of the decay interaction. The comparison of the rate for $K_2{}^0 \to 3\pi$ with $K^+ \to 3\pi$ provides a test[10,11] of the $\Delta I = \frac{1}{2}$ rule in nonleptonic decay, which also requires that the energy spectrum of the neutral pion in decay (3) should be the same as that of the charged pion in τ' decay.[11,12]

The current of strongly interacting particles in K decay can carry an isotopic spin change of $\Delta I = \frac{1}{2}$ or $\Delta I = \frac{3}{2}$. It has often been suggested that the $\Delta I = \frac{3}{2}$ currents may be absent.[13] This is sometimes called the leptonic $\Delta I = \frac{1}{2}$ rule. Transitions with $\Delta S/\Delta Q = -1$, where S and Q are the strangeness and charge of the strongly interacting particles involved, require $\Delta I = \frac{3}{2}$ currents, although these currents may exist even if $\Delta S/\Delta Q = -1$ transitions are forbidden.[14] Such transitions must be at least approximately forbidden to explain the absence of $\Xi \to n + \pi^-$ decays, in the simplest version of the "current times current" theory of weak interactions.[13] There are now indications[14a] that the $\Delta S = \Delta Q$ rule is violated.[15–17] Independently of the $\Delta S = \Delta Q$ rule, the presence of $\Delta I = \frac{3}{2}$ currents can be investigated by comparing the rates for $K_2{}^0$ leptonic decay, decays (1) and (2), with the rates for the corresponding K^+ leptonic decays.[13,14]

The experimental exploitation of these proposed measurements has not proceeded very far, essentially because of the impurity, low intensity, and energy spread of the available $K_2{}^0$ beams, and because of the difficulty in identifying the products of $K_2{}^0$ decay. Two experiments similar to that of Ref. 1 have given further data on the $K_2{}^0$ branching ratios,[5,18] and an experiment on the interactions and decays of high-energy $K_2{}^0$ mesons has been performed.[19] The experiments on the three-body decays of K^0 mesons initially of definite strangeness, also provide information on $K_2{}^0$ decays, from those particles which decay several $K_1{}^0$ lifetimes from the point of production.[15,16]

This paper describes an experiment performed at the Cosmotron with a low-energy $K_2{}^0$ beam in a hydrogen bubble chamber. Results were obtained on a number of the points mentioned above. Some preliminary results on the decays and the interactions in hydrogen have been presented in a previous communication.[20]

II. BEAM

The $K_2{}^0$ beam was planned to be of the lowest practical momentum for two reasons. First, the interest in the $K_2{}^0 - P$ interactions lies in a comparison with the $K^+ - P$ and $K^- - P$ data in a region where the latter two have been carefully investigated, that is, from zero to $\sim$400 MeV/c. Second, the method used for the identification of the $K_2{}^0$ decay products, measurement of bubble density, is applicable only for low momentum particles.

The $K_2{}^0$ beam used in this experiment was made by (1230±60) MeV/c π^- mesons striking a polyethylene target. The $K_2{}^0$ mesons produced at an angle of $47\frac{1}{2}° \pm 2°$ were accepted by the beam channel.

The K^0's were produced by the reactions

$$\pi^- + N \to K^0 + \Sigma \tag{5}$$

$$\pi^- + P \to K^0 + \Lambda . \tag{6}$$

The K^0's from reactions (5) and (6) have momentum distributions centered at 300 and 520 MeV/c. It would be possible to choose a π^- momentum and a K^0 angle such that only K^0's from reaction (6) would be accepted; however, if it is attempted to create K^0's of less than 400 MeV/c in this way, the K^0 angle in the $\pi^- + P$ center of mass will be near 180°, where the cross section is extremely small. Since the cross section for reaction (5) is roughly isotropic in the c. m. system, on the other hand, it is suitable for producing K^0's of low momentum. The K^0's accepted were produced near the limiting angle in the laboratory.

The charged particles were removed from the beam by a 36-in. sweeping magnet (Fig. 1). The 6-in. gap of this magnet was filled with brass and lead except for a channel defining the $K_2{}^0$ beam. Near the end of this channel there was a 3.5 cm-thick lead filter to reduce the number of γ rays in the beam. The 20-in. hydrogen bubble chamber was placed immediately after the sweeping magnet, with another 12 in. of lead shielding in the gap of the bubble chamber magnet. The center of bubble chamber was 120 in. from the target.

The target was shaped to fill the parallelpiped defined by the intersection of the $K_2{}^0$ beam and the π^- beam. It was 7 in. long in the direction of the π^- beam, 5 in. in the direction of the $K_2{}^0$ beam, and 4 in. high. The pions which traverse the full length of the target lose 45

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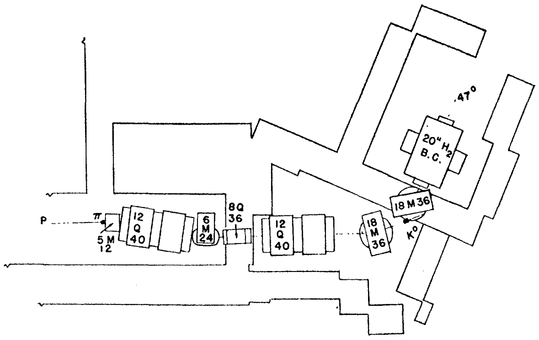

FIG. 1. The experimental arrangement at the Cosmotron. The target where the pions are produced by the external proton beam is marked π, while K^0's are produced at the target marked K^0. The titles on the beam magnets give the aperture, nature of the magnet (quadrupole or bending magnet) and length, in inches. The outlined areas denote steel and concrete shielding.

MeV/c; the central value and width of the pion momentum quoted above took into account this effect.

The π^- beam was similar to the beam used in the experiment of Ref. 19 with small modifications. The effective solid angle was about 5 msr. The pions were created by the 2.85-BeV protons in the external beam from the Cosmotron, focussed on a $2\times2\times4$ in.-long brass target. The pion intensity was measured with an ionization chamber calibrated by comparison with nuclear emulsion plates. An average external proton beam of 3×10^{10} protons per pulse resulted in 1.5×10^{6} π^- per pulse, at the K^0 target.

This number of pions gave $\sim\frac{2}{3}$ $K_2{}^0$ passing through the fiducial volume of the bubble chamber. The background consisted of: a number of short proton tracks, caused by neutron collisions; several energetic recoiling protons; a few low-energy electron-positron pairs; and a few tracks of pions which scattered in the sweeping magnet and managed to reach the bubble chamber. Three-prong neutron stars and energetic electron pairs were relatively rare. It was verified that the background was greatly reduced when the polyethylene target was removed.

III. SCANNING AND MEASUREMENT

About 60 000 pictures of good quality were obtained. The pictures were scanned for events of two types: single V^0 events not associated with a production vertex; and $K_2{}^0$ interaction with hydrogen, such as $K_2{}^0+P\rightarrow\Lambda+\pi^+$ or $K_1{}^0+P$. It is easy to separate the decays and interactions except for those cases of $K_1{}^0+P$ where the proton has a range of less than one millimeter. Extrapolation of the measured differential cross section shows that less than one event of this type is expected. Contamination of the decays by other events which are not V^0's but have a similar appearance will be discussed in the next section. Events were accepted only if the decay vertex was in a fiducial volume of 13.6 liters. (The visible volume of the chamber is $\sim$30 liters.) With this condition, most tracks which had to be measured were $\gtrsim$10 cm in length. Rescanning of 20% of the pictures was carried out during the primary scan, and the scanning efficiency separately determined for the common types of events. The single scan efficiency was $(93\pm3)\%$ for all $K_2{}^0$ decays. There is no significant difference in efficiency among the various event types; therefore, since only ratios of the cross sections for different reactions will be discussed in this paper, no correction need be made for scanning efficiency.

The events were measured with digitized measuring machines and the tracks reconstructed by means of the computer program TRED. The bubble density for each track in the event and for reference tracks was measured by recording the length of each gap with the measuring machines, and applying the maximum likelihood formula for the projected bubble density. This number was corrected for dip and conical projection and compared to fast π or μ tracks in the same picture. These measurements were usually repeated by a second observer to ensure that consistent results were obtained.

IV. IDENTIFICATION OF EVENTS

It has become customary to characterize the kinematic analysis of a type of event by the number of kinematic constraints, which is defined as the number of equations expressing energy and momentum balance at the interaction vertex minus the number of unmeasured quantities. For an interaction initiated by a charged particle with all the outgoing particles measured, there are four constraints, and this is called a 4-c case.

In the present experiment, the direction of the incoming particle is known with a small error, but the momentum is unknown, within a wide range. If all the outgoing particles from a vertex are measured, the event will be of the type 3-c. Events from the interactions $K_2{}^0+P\rightarrow\Lambda+\pi^+$ and $K_1{}^0+P$ are of this type, since they are considered only if the V^0 is seen. In the $K_2{}^0$ energy range of the present experiment, identification of the constrained events presented no problem.

The events with three constraints were used to check the distribution of incoming $K_2{}^0$ directions calculated from the geometry of the beam. The fraction of these events from $K_2{}^0$ which did not enter the chamber with the beam direction was $<10\%$.

Decays

Since each of the three visible $K_2{}^0$ decay modes has a neutral particle, all are of the zero constraint type. The measurements of momenta and angles are, in general, consistent with all three decay modes. Moreover, the presence of a square root in the kinematic equations for the 0-c case leads to two different solutions, corresponding to different $K_2{}^0$ momenta. This equation may be written:

$$P_0=\frac{P_{\parallel}(M_0{}^2-M_1{}^2+M_c{}^2)\pm U_c[(M_0{}^2-M_1{}^2-M_c{}^2)^2-4M_1{}^2M_c{}^2-4M_0{}^2P_\perp{}^2]^{1/2}}{2(P_\perp{}^2+M_c{}^2)}, \tag{7}$$

where $P_0\equiv$momentum of $K_2{}^0$; (velocity of light$=1$) $P_{\parallel}\equiv$component of P_c along the line of flight of $K_2{}^0$ (which is a known direction), where P_c is the vector sum of the momenta of the two charged decay products; $P_\perp\equiv$component of P_c in the plane perpendicular to the $K_2{}^0$ line of flight; M_0=mass of $K^0=497.8$ MeV; M_1 =mass of the neutral decay product; M_c= effective mass of the two charged decay products; and $U_c=(P_c{}^2+M_c{}^2)^{1/2}$. Consequently, there are in general ten possible interpretations of a given event, and it is necessary to devise a technique for selecting the correct one, in at least a well-defined subset of events. This was possible, in the present experiment, for decay (1), while the rate of events from decay (3) was determined by a statistical study of the remaining events. The rate for decay (2) is then determined by subtraction of (1) and (3) from the total number of events.

Events representing decay (1) were identified if the electron had a momentum of less than 200 MeV/c. The electrons were recognized by bubble density measurements[21] on all the decay tracks which were not obviously densely ionizing. Since about one-third of the electrons from this reaction have momenta greater than 200 MeV/c, the identified events form a biased sample. However, the evaluation of the number of events of given configuration which fail to be identified is straightforward. (Two variables are required to specify the configuration of the decay, if the polarizations of the leptons are not measured. We have chosen to use the kinetic energies of the electron and pion, T_e and T_π.) Consider a $K_2{}^0$ of known momentum, decaying by the $(\pi e\nu)$ mode. Since the K^0 has no spin, the angular distribution of each of the decay particles is isotropic in the rest system of the $K_2{}^0$. Electrons of a certain energy T_e will have different momenta in the laboratory, depending on the angle they form with the $K_2{}^0$ direction in the c.m. system, θ_c. For small T_e, the laboratory momentum is less than 200 MeV/c for all angles, and all such decays are identified. As T_e is increased, the laboratory momentum of electrons emitted in the forward direction is greater than 200 MeV/c, and these will not be identified.

If electrons emitted at an angle $\theta_c(T_e)$ have a laboratory momentum=200 MeV/c, the fraction of events identified is just the fraction of solid angle beyond θ_c, or

$$F_{\mathrm{ID}}=\tfrac{1}{2}+\tfrac{1}{2}\cos(\theta_c(T_e)).$$

This "identification efficiency" must be averaged over the momentum spectrum of the $K_2{}^0$'s, but it is not very sensitive for the $P_{K_2{}^0}$ of the present experiment. The average was made using the spectrum obtained experimentally, given in Sec. XIII.

[21] A detailed description of bubble density measurements made in the same bubble chamber is given in B. N. Fabian, R. L. Place, W. A. Riley, W. H. Sims, and V. P. Kenney, Rev. Sci. Instr. 34, 484 (1963).

Since there was no kinematic check on the identification of an individual event, several precautions were taken to eliminate possible sources of background from physical processes other than $K_2{}^0$ decay. Any event which fell into one of the following classes was rejected:

(1) Opening angle of the V^0 less than 5° or greater than 165°. The first condition is effective in eliminating high-energy electron pairs, where it is difficult to identify both particles as electrons.

(2) Both particles identified as electrons.

(3) Event interpretable as $\pi^\pm\rightarrow\mu^\pm+\nu$, taking into account the expected ranges of the particles.

(4) Event interpretable as $\pi^\pm+p\rightarrow\pi^\pm+p$, with a proton range less than one millimeter.

(5) Event interpretable as $\mu^\pm\rightarrow e^\pm+\nu+\nu$, for μ momenta less than 100 MeV/c.

(6) Events with one particle having a range of less than one centimeter.

(7) Events with one particle identified as an electron of less than 20 MeV/c.

(8) Events with a track with a dip angle of greater than 70°. This requirement is dictated by the necessity of bubble density measurements.

These restrictions reduce the background events to a negligible number. For example, the number of $\mu^\pm-e^\pm+\nu+\nu$ decay in flight, the most dangerous of the background processes, was estimated by counting the number of μ decays at rest, and determining the flux and momentum spectrum of the background tracks entering the chamber. After applying restriction number five above, one calculates that less than one event would be falsely identified as a $K_2{}^0$.

The number of genuine $K_2{}^0$ decays from (1), (2), and (3) which were eliminated by these restrictions was determined by a method very similar to that used for calculating the $(P_0')^2$ distribution described in Sec. VI. Events were generated by the computer and tested to see if one of the above conditions was fulfilled, within the errors of a typical real event. The range of the kinematic variables was covered systematically with a sufficient number of "events" so that a value for the fraction of true events lost was obtained for a given event configuration, with an absolute accuracy of about 2%. The number of true events lost, averaged over the event configurations, was only of the order of 4 to 5% for each of the three event types. [The fractions retained have been introduced into Eqs. (13) and (14).] A plot of the probability of the identification by bubble

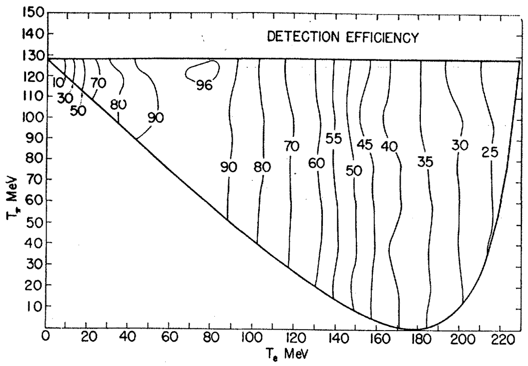

FIG. 2. The probability, in percent, of detection and identification of ($\pi e\nu$) decays is shown as a contour plot covering the allowed region in T_e and T_π, the kinetic energy of the electron and pion.

density measurements of an event from (1), times the probability that it will not be rejected by one of the restrictions listed, is shown in Fig. 2, in the form of contours of equal probability, for different event configurations specified by T_e and T_π.

The number of misidentified events and the effects of the ambiguities in the identification of ($\pi e\nu$) events on the distribution of event configurations are described in the Appendix.

V. RESULTS FOR $K_2^0 \to (\pi e\nu)$

A. Test of *CP* Invariance

The 153 events which are identified as ($\pi e\nu$) decays are shown on an energy scatter-plot in Fig. 3. Of these, 76 are ($\pi^- e^+\nu$) and 77 are ($\pi^+ e^-\nu$). The ratio

$$(\pi^- e^+\nu)/(\pi^+ e^-\nu) = 0.99 \pm 0.16 \qquad (8)$$

is consistent with the value unity requi: ed by *CP* conservation,[22] in agreement with previous experiments.[2,23]

B. The Decay Interaction

If it is assumed that the (e,ν) interaction is local: that only the sum of the e and ν momenta is relevant, the

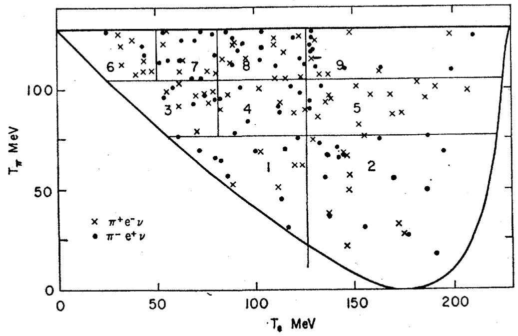

FIG. 3. The identified ($\pi e\nu$) events on a scatter plot in T_e and T_π.

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distribution of ($\pi e\nu$) events on the energy plot, a scatter plot with coordinates (T_π, T_e), is given uniquely by the form of the decay interaction,[9] except for a form factor which is a function of T_π. This factor depends on the effects of strong interactions. Consequently, this distribution may be written in the form

$$d^2W/dT_\pi dT_e = f^2(T_\pi) W_D(T_e, T_\pi) \qquad (9)$$

where D may be S, V, or T; for scalar, vector and tensor interactions.[9] (We will not consider combinations of different interactions.) It is expected that $f^2(T_\pi)$ will be approximately constant over the range of T_π, $0 \leq T_\pi \leq 130$ MeV.[24]

The functions W_S, W_V, and W_T, in the form of contours of constant $d^2W/dT_\pi dT_e$, are given in Fig. 4. It is evident that these functions reduce to different functions of T_e for a fixed value of T_π. This fact allows a comparison of the data with the different forms of W_D to be made which is independent of $f^2(T_\pi)$, although we will show that the T_π variation of $W_D(T_e, T_\pi)$ is a more sensitive test, if $f^2(T_\pi) \simeq$ constant. Indeed, in most of the nuclear "neutrino-correlation" experiments it is the recoil-momentum distribution function which distinguishes among the various possible interactions, which amounts to measuring the form factor and choosing the interaction which makes the form factor most nearly constant.[25] This is the situation in this experiment, except that the maximum momentum transfer is larger, allowing the possibility of an appreciable variation in the true form factor, and that it is possible to eliminate one of the types of interaction by means of the electron spectrum alone. It will be shown that when the form factor is assumed to be constant only one type of interaction is consistent with the distribution in T_π.

To proceed in a manner essentially independent of form factor variation, the T_π axis is divided into three segments, and the area of the energy plot is subdivided into the nine regions in T_π and T_e shown in Fig. 3. If the three $W_D(T_e, T_\pi)$ are integrated over each of these regions, it is easy to see that a variation of $f^2(T_\pi)$ within one of the regions in T_π will change the T_e dependence of the integrated W_D values only slightly. In order to compare directly with the identified ($\pi e\nu$) cases, the quantity actually integrated over the regions on the energy plot was W_D times the probability of detection and identification given as a function of T_e and T_π in Fig. 2.

This procedure gives the relative number of events expected to be found in each of the (T_e, T_π) regions on the assumption of a given type of interaction and a constant form factor. To make a comparison with the data, the sum of the integrals over the nine energy regions and the number of events observed are each normalized to 100%, as shown in Fig. 5(a). This comparison is valid only if the form factors are slowly vary-

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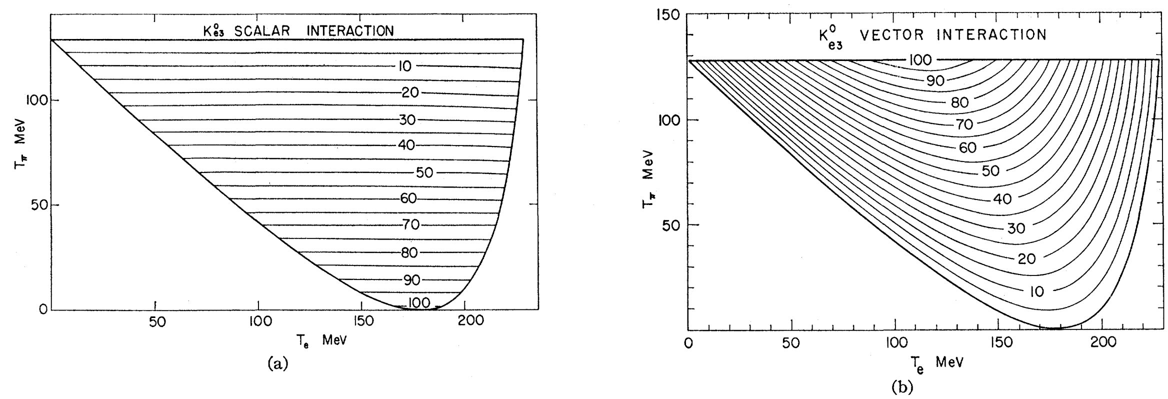
[25] O. Kofed-Hansen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **28**, No. 9 (1954).



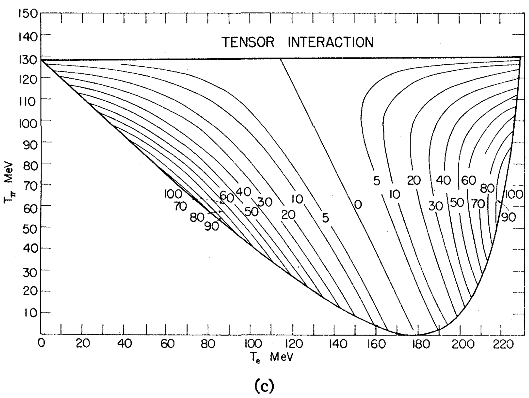


FIG. 4. A contour plot of the matrix element squared for $(\pi e\nu)$ decay for a constant form factor and: (a) scalar interaction, (b) vector interaction, and (c) tensor interaction.

ing. The effect of a possible variation of the form factor can be essentially eliminated by normalizing the number of events and the theoretical distributions to 100% in each of the three segments of T_π; for example, in regions 6, 7, 8, and 9. The result of this procedure is shown in Fig. 5(b). The errors shown are the statistical errors on the number of events.

In Fig. 5(a), the scalar and tensor interactions are in very bad agreement with the experimental results, while the vector interaction fits the data reasonably well. The comparison shown in Fig. 5(b), which is not sensitive to the form factor variation, is statistically less powerful. The tensor interaction is clearly in poor agreement with the data, and this is confirmed by a χ^2 test. Either the vector or the scalar interaction is consistent with the data. The variation of the form factor required to make the scalar interaction fit the experiment will be examined in the next section.

C. Form Factor

The average values of the form factor in each of the three intervals of T_π is given by the number of events in an interval, divided by the product $W_D(T_e,T_\pi)F_{\mathrm{ID}}(T_e,T_\pi)$ integrated over the appropriate region in (T_e,T_π).

The results, shown in Fig. 6, are very different for the three types of interaction. In fact, if $f^2(T_\pi)$ may be assumed to be approximately constant, these plots provide a determination of the decay interaction which is statistically much stronger than that from the T_e spectra. It is at once apparent from Fig. 6 that *only the vector interaction gives a slowly varying form factor.* The $f_S{}^2(T_\pi)$ in the scalar case changes by a factor of $\sim$14 between the first and third intervals in T_π, and an inspection of the energy plot shows many events with T_π near the upper limit, where $W_S(T_e,T_\pi)$ goes to zero. The true variation in $F_S{}^2(T_\pi)$ must be very strong near this upper limit, a circumstance so improbable that we may confidently exclude the pure scalar interaction.

The vector form factor points are consistent with a constant or slightly decreasing value. This result is in accord with theoretical expectation as illustrated by the curves drawn on Fig. 6(a) showing the variation expected if the interaction is mediated by an intermediate vector boson.[24] A similar variation arises from the existence of a P-wave $K\pi$ resonance, and since there is such a resonance[26] at 885 MeV mass, the variation in f^2 could be at least as strong as that corresponding to $M_B=885$.

D. Average Identification Probability

The measured $f_V{}^2(T_\pi)$ can be used, together with the function $W_V(T_e,T_\pi)$, to form the weighted average of

[26] R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, C. Ghesquière, B. P. Gregory, D. Rahm, P. Rivet, and F. Solmitz, in *Proceedings of the 1962 Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 295.

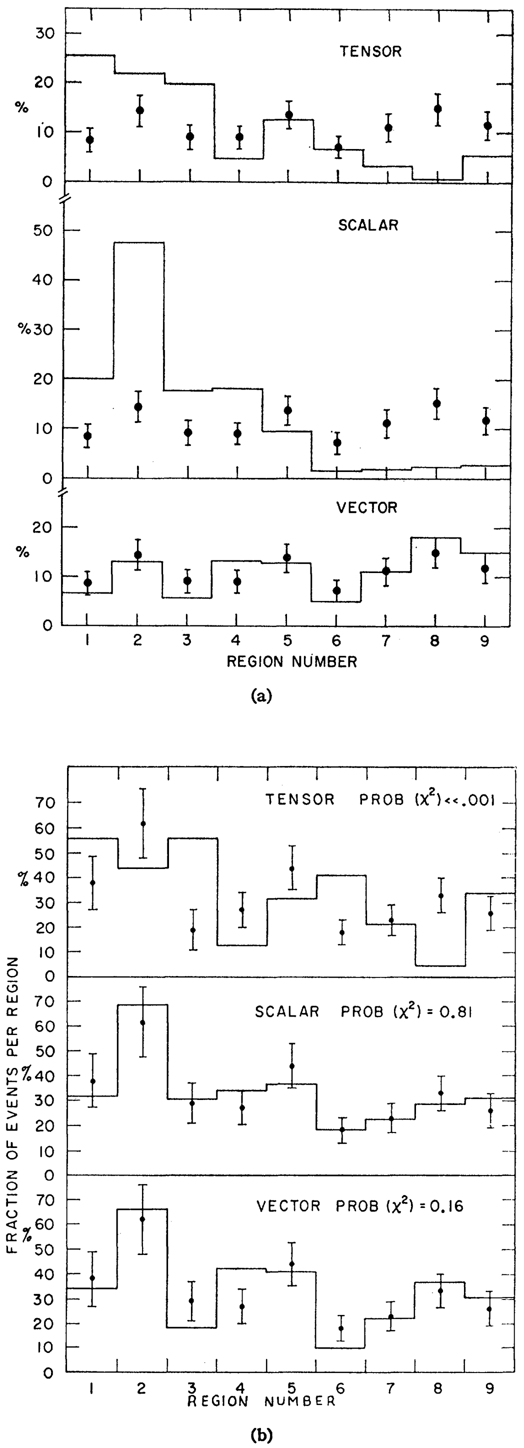


FIG. 5. A comparison of the $(\pi e\nu)$ events with the S, V, and T matrix elements. The histograms represent the theoretical distribution of events, corrected for detection efficiency and normalized in each set of regions covering the same interval of T_π. The points are the experimental fractions of events in those regions. (a) The number of events in the whole plot is normalized to 100%. (b) The number of events in each segment of T_π is normalized to 100%, eliminating the effects of a possible variation of the form factor.

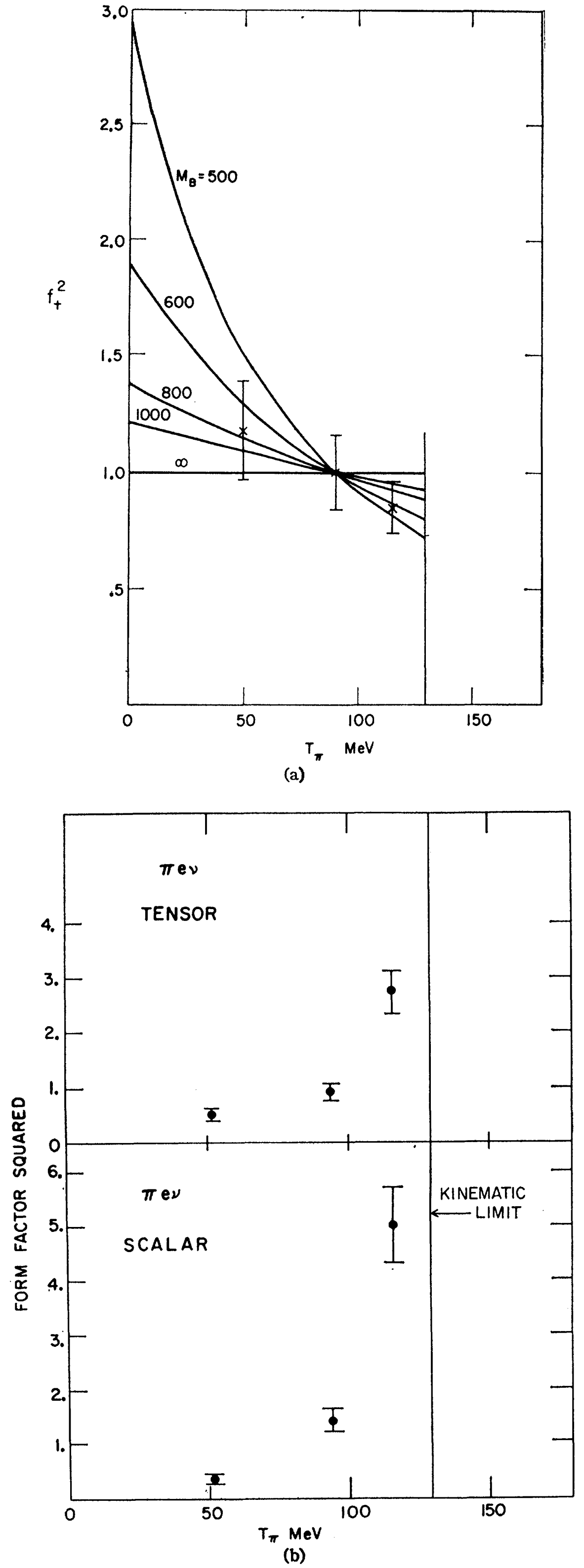


FIG. 6. The form factor in three energy regions, for: (a) the vector interaction, (b) the scalar interaction and the tensor interaction.

$F_e(T_\pi, T_e)$ integrated over the T_e and T_π variables. The result is that 0.62 of all $(\pi e\nu)$ decays are accepted and identified, while the probability for accepting an event with or without identification is 0.95. These numbers will be used in calculating the branching ratios.

VI. $K_2{}^0 \to \pi^+\pi^-\pi^0$

It is not possible to identify a useful fraction of the remaining decays on the basis of measurements on individual events. However, the energy release in $(\pi^+\pi^-\pi^0)$ decay is much smaller than in the other two decay modes. This results not only in a limit on the effective mass of the two visible prongs, but also in a strong limitation on the visible transverse momentum with respect to the $K_2{}^0$ line-of-flight. These limitations are contained in the formula for the $K_2{}^0$ momentum, Eq. (1); an event which is inconsistent with $(\pi^+\pi^-\pi^0)$ decay will give an imaginary square root in that formula. We will show that essentially all the unidentified $(\pi e\nu)$ decays, and all but a small fraction of the $(\pi\mu\nu)$ decays are in fact inconsistent with the hypothesis of $(\pi^+\pi^-\pi^0)$ decay.

Inspection of Eq. (7) shows that the existence of a real solution for the $K_2{}^0$ momentum depends only on $P_\perp$. Consequently, it is useful to consider quantity $(P_0')^2$ given by Eq. (7) when $P_{\parallel}$ is set equal to zero[27]:

$$(P_0')^2 = \frac{(M_0{}^2 - M_1{}^2 - M_c{}^2)^2 - 4M_1{}^2M_c{}^2 - 4M_0{}^2P_\perp{}^2}{4(P_\perp{}^2 + M_c{}^2)}. \quad (10)$$

$(P_0')^2$ is an invariant in the sense that the distribution in $(P_0')^2$ is independent of the $K_2{}^0$ momentum. An event consistent with the hypothesis characterized by the masses assigned to the decay particles must have $(P_0')^2 \geqslant 0$ as well as

$$M_0 \geqslant M_1 + M_c.$$

The expected distributions in $(P_0')^2$ for the three decay modes have been computed by generating events of each type on a computer, and then computing $(P_0')^2$ with $(\pi^+\pi^-\pi^0)$ masses. The configurations for the leptonic decays were distributed according to the vector interaction matrix element; the $(\pi^+\pi^-\pi^0)$ decays were distributed with uniform density on the Dalitz plot. For a given configuration, the remaining variables are equivalent to different orientations of the particles in the $K_2{}^0$ rest system. The distribution was integrated over the four variables corresponding to the configuration and orientation with about 10 000 points for each decay mode. Configurations of $(\pi e\nu)$ which gave electrons with momentum in the laboratory less than 200 MeV/c were excluded from this integration, since real events of this sort are identified and excluded from the experiment histogram of $(P_0')^2$, shown in Fig. 7. The calculated experimental resolution was folded into the resulting distributions, which are also shown in Fig. 7, with a relative normalization described below.

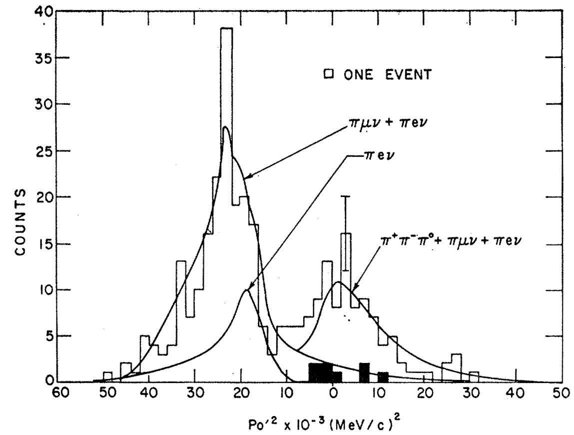


FIG. 7. The distribution of the quantity $(P_0')^2$ for all events except those with identified electrons.

The histogram in Fig. 7 contains all the events except those identified as $(\pi e\nu)$ decays. The area under the three theoretical $(P_0')^2$ distributions is normalized to the same number of events. Moreover, the number of nonidentified $(\pi e\nu)$ events is known, from the number of identified $(\pi e\nu)$ events and the probability of identification quoted in Sec. IV. There remains one parameter: the relative proportion of $(\pi\mu\nu)$ and $(\pi\pi\pi)$ events, which was fixed by the requirement that the curves be separately normalized for $(P_0')^2 \geq -5000$ MeV/c^2 and $(P_0')^2 < -5000$ MeV/c^2. This procedure determines the $(\pi\mu\nu)/(\pi^+\pi^-\pi^0)$ ratio relatively well, since these types of events are well separated in $(P_0')^2$. Less than 10% of the $(\pi\mu\nu)$ events have $(P_0')^2 > -5000$ and, of course, the $(\pi^+\pi^-\pi^0)$ events with $(P_0')^2 < 0$ are only due to the experimental resolution. The result is that 0.23 ± 0.03 of the unidentified events are $(\pi^+\pi^-\pi^0)$ decays. The curves normalized with this one free parameter are seen to constitute a satisfactory fit to the experimental histogram. In particular, the two peaks predicted for the leptonic and pionic decays are clearly visible.

Some additional confirmation of this picture is provided by considering the values of $K_2{}^0$ momentum [on the $(\pi^+\pi^-\pi^0)$ hypothesis] for the cases with $(P_0')^2 > -5000$. The events for which neither of the P_0 values are in the bounds $200 \leqslant P_0 \leqslant 675$ MeV/c which include 95% of the $K_2{}^0$ momenta, are shaded in the histogram of Fig. 7. The program used to calculate the $(P_0')^2$ distribution of the $(\pi\mu\nu)$ events also indicated that those which yielded solutions for $(\pi^+\pi^-\pi^0)$ decay would, on that hypothesis, give a momentum very different from the true $K_2{}^0$ momentum. The shaded part of the histogram is to be compared with the $(P_0')^2$ curve for the $(\pi\mu\nu)$ decays. The satisfactory agreement indicates that

[27] The distribution of an equivalent quantity was used in Ref. 18.

the unshaded part of the histogram consists of almost entirely of the $(\pi^+\pi^-\pi^0)$ decays.[28]

VII. BRANCHING RATIOS

The analysis in the preceding sections provides the basis for the calculation of a complete set of branching ratios for K_2^0 decay into charged particles, on the assumption that only the three known decay modes are significant. The quantities to be determined are: $N_t \equiv$ total number of K_2^0 decays into charged particles; $R_{\pi e\nu}$, $R_{\pi\mu\nu}$, $R_{\pi^+\pi^-\pi^0} \equiv$ fractions of N_t decaying into the channels indicated. These four quantities are connected by four equations:

$$R_{\pi e\nu}+R_{\pi\mu\nu}+R_{\pi^+\pi^-\pi^0}=1\,, \tag{11}$$

$$N_{\mathrm{ID}}=0.62N_tR_{\pi e\nu}\,, \tag{12}$$

$$N_{\mathrm{NID}}=0.95N_tR_{\pi\mu\nu}+0.93N_tR_{\pi^+\pi^-\pi^0}+(0.95-0.62)N_tR_{\pi e\nu}\,, \tag{13}$$

$$F_{3\pi}N_{\mathrm{NID}}=0.93N_tR_{\pi^+\pi^-\pi^0}\,, \tag{14}$$

where N_{ID} = number of identified $(\pi e\nu)$ decays, N_{NID} = the number of K_2^0 decays which pass the acceptance criteria and which do not have identified electrons, $F_{3\pi}$ = the fractions of the N_{NID} cases arising from $(\pi^+\pi^-\pi^0)$ decay, as determined from the $(P_0')^2$ distribution, Sec. VI. The factors appearing in the above equations are the efficiency factors from Sec. IV and V. The experimental values for the above quantities are:

$$N_{\mathrm{ID}}=153\,, \tag{15}$$

$$N_{\mathrm{NID}}=326\,, \tag{16}$$

$$F_{3\pi}=0.24\pm0.035\,. \tag{17}$$

The solution of the equations then yields the branching ratios given in Table I. The errors are obtained from statistics and the quoted systematic error in $F_{3\pi}$. The ratio of $(\pi\mu\nu)$ to $(\pi e\nu)$ decays is given separately, since the branching ratio errors are highly correlated.

Also given in Table I are the predictions obtained from K$^+$ branching ratios if the $\Delta I=\frac{1}{2}$ rule is used to predict the $(\pi^+\pi^-\pi^0)$ rate and either $\Delta I=\frac{1}{2}$ or $\Delta I=\frac{3}{2}$ currents of strongly interacting particles are assumed to dominate in the leptonic decays. (The details of these calculations are given in the Sec. VIII.)

It is seen that the experimental values are in good agreement with the predictions of the $\Delta I=\frac{1}{2}$ current, while they are in very bad agreement with the predictions of the $\Delta I=\frac{3}{2}$ current. The results of this experiment alone would tend to support the $\Delta I=\frac{1}{2}$ rule and $\Delta I=\frac{1}{2}$ currents in the leptonic decays; however, the possibility exists that both fail in such a way that the ratios remain correct. This will be discussed in Sec. VIII where our branching ratios are combined with the results of other experiments to test these rules separately.

The Ratio $(\pi\mu\nu)/(\pi e\nu)$

The experimental value for the ratio of $(\pi\mu\nu)$ to $(\pi e\nu)$,

$$R=R(\pi\mu\nu)/R(\pi e\nu)=0.73\pm0.15\,, \tag{18}$$

may be compared with the ratio of phase space in the two decays, which is 0.65. This ratio can deviate from 0.65, however, even if μ-e universality holds. This comes about in the following way. There are two independent vectors which may be formed from P_K and P_π. It is advantageous to express the matrix element in $(\pi L\nu)$ decay in terms of their sum and difference,

$$M_{(\pi L\nu)}\propto(P_K+P_\pi)f_+(T_\pi)+(P_K-P_\pi)f_-(T_\pi)\,. \tag{19}$$

Then it is found[9,24] that when the decay distribution is expressed in terms of T_L and T_π, the second term is multiplied by $(M_L/M_K)^2$. In practice this term is negligible for $(\pi e\nu)$ decay, but it may be appreciable for $(\pi\mu\nu)$. Consequently, the form factor given in Fig. 5 is $f_+(T_\pi)$. If $f_+(T_\pi)$ and $f_-(T_\pi)$ are constant, one finds that[9]

$$R=0.65+0.124\xi+0.019\xi^2\geqslant0.46,\ \text{where } \xi=f_-/f_+\,. \tag{20}$$

Using our experimental value of R, we obtain two solutions for ξ:

$$\xi=0.66_{-1.3}^{+0.9}\quad\text{or}\quad-6.6_{-1.5}^{+0.7}\,; \tag{21}$$

the first is consistent with $\xi=0$.

Several authors have developed dispersion relations for calculating f_+ and f_-. In general, ξ is expected to be small:

$|\xi|<0.3$ (MacDowell)[24]
$\xi\simeq0.2$ (Bernstein and Weinberg),[29]

in agreement with one of our solutions. The values of R obtained in K^+ decay, given in Table I, are somewhat larger than, but not in significant disagreement with, our value. A difference in these values would indicate the presence of $\Delta I=\frac{3}{2}$ currents in at least one of the decays.

VIII. TESTS OF THE $\Delta I=\frac{1}{2}$ Rule

The $\Delta I=\frac{1}{2}$ rule in nonleptonic strange particle decays is in agreement with almost all the experimental results.[30] (The asymmetries in Σ decay[31] are a possible exception.) The rule that only $\Delta I=\frac{1}{2}$ currents (of strongly interacting particles) interact in the leptonic decays of strange particles is sometimes called the leptonic $\Delta I=\frac{1}{2}$ rule. There are not many tests of this

[28] In order to obtain the whole $(\pi^+\pi^-\pi^0)$ sample it is necessary to include the events with slightly negative $(P_0')^2$. The same technique may still be used to separate the $(\pi^+\pi^-\pi^0)$ events from the $(\pi\mu\nu)$ contamination with the same $(P_0')^2$ since the imaginary part of P_0 is small ($\lesssim$70 MeV/c), and can be ignored.

[29] J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960).

[30] L. B. Okun, *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 743.

[31] R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **9**, 66 (1962).

TABLE I. $K_2{}^0$ branching ratios measured in this experiment, together with the predictions made from K^+ rates using the $\Delta I=\frac{1}{2}$ rule and $\Delta I=\frac{1}{2}$ or $\frac{3}{2}$ currents applied together.

Branching ratio	This experiment	Predictions from K^+ decay $\Delta I=\frac{1}{2}$ currents		$\Delta I=\frac{3}{2}$ currents	
$R(\pi e\nu)$	0.487 ± 0.05	0.433 ± 0.043[a]	0.456 ± 0.042[b]	0.285 ± 0.035[a]	0.306 ± 0.028[b]
$R(\pi\mu\nu)$	0.356 ± 0.07	0.415 ± 0.052	0.383 ± 0.038	0.297 ± 0.029	0.257 ± 0.026
$R(\pi^+\pi^-\pi^0)$	0.157 ± 0.03	0.152 ± 0.017	0.163 ± 0.011	0.417 ± 0.047	0.437 ± 0.030
$R(\pi\mu\nu)/R(\pi e\nu)$	$0.73\ \pm0.15$	$0.96\ \pm0.15$	$0.84\ \pm0.11$	$0.96\ \pm0.15$	$0.84\ \pm0.11$

[a] Computed from rates given under Ref. a in Table III.
[b] Computed from rates given under Ref. b in Table III.

rule, and it seems to have failed in some cases where tests were possible.[32]

However, it was pointed out in the last section that the results of this experiment are in agreement with the predictions made by assuming both rules are valid, while they would not be consistent with the values obtained by assuming the validity of the $\Delta I=\frac{1}{2}$ rule, and, for example, equal admixtures of $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ currents.

The relations used to obtain the predictions in Table I are[10,11,14]

$$\Gamma_{2(+-0)}=2\Gamma_{+(++c)}(1.28/1.24) \tag{22}$$

and

$$\Gamma_{2(L)}=2\Gamma_{+(L)}(\Delta I=\tfrac{1}{2}\text{ current}) \tag{23}$$

or

$$\Gamma_{2(L)}=\tfrac{1}{2}\Gamma_{+(L)}(\Delta I=\tfrac{3}{2}\text{ current}), \tag{24}$$

where $\Gamma_{2(+-0)}$ is the $K_2{}^0$ decay rate into the $(\pi^+\pi^-\pi^0)$ mode, $\Gamma_{+(++-)}$ is the K^+ decay rate into the $(\pi^+\pi^+\pi^-)$ mode, etc.; and

$$\Gamma_{2(L)}=\Gamma_{2(\pi^\pm e^\mp\nu)}+\Gamma_{2(\pi^\pm\mu^\mp\nu)}.$$

The last factor in the first equation corrects for the different phase space volumes due to the $(K^+-K_2{}^0)$ and $(\pi^+-\pi^0)$ mass differences.[10] The phase space volumes for the four possible $K\to3\pi$ decays are given in Table II. This factor is nearly equal to one for the leptonic decays. The decay rates in K+ decay are given in Table III, for the bubble chamber experiment of Roe *et al.*[33] and for the average of results of that experiment with a number of emulsion experiments.[34]

TABLE II. The maximum kinetic energy of the "odd" pion, $T_{\max}$, and the integral of the phase space volume V for the four $K\to3\pi$ decay modes, using $M_{K^+}=493.9$ MeV, $M_{K^0}=497.8$ MeV, $M_{\pi^+}=139.6$ MeV, $M_{\pi^0}=135.0$ MeV.

Decay	$T_{\max}$	V
$K^0\to\pi^0\pi^0\pi^0$	58.98 MeV	1.566
$K^+\to\pi^+\pi^0\pi^0$	53.26 MeV	1.244
$K^0\to\pi^+\pi^-\pi^0$	53.90 MeV	1.284
$K^+\to\pi^+\pi^+\pi^-$	48.16 MeV	1.000

A direct test of the $\Delta I=\frac{1}{2}$ rule may be made by comparing $\Gamma_{2(000)}/\Gamma_{2(+-0)}$ with the value predicted from the K^+ data[10,11]:

$$\frac{\Gamma_{2(000)}}{\Gamma_{2(+-0)}}=\left(\frac{1.57}{1.28}\right)\left(\frac{1}{2}\frac{(1.24)\Gamma_{+(++-)}}{(1.00)\Gamma_{+(+00)}}-\frac{1}{2}\right)=2.00\pm0.19. \tag{25}$$

This equation is valid for any combination of the pion states (with $I=1$) with different symmetries. [If the symmetrical state dominates,

$$\Gamma_{2(000)}/\Gamma_{2(+-0)}=(1.57/1.28)(\tfrac{3}{2})=1.84.] \tag{26}$$

This ratio may be obtained by combining our data with Anikina *et al.*[5] who have measured the fraction of $K_2{}^0$ decaying into (presumably three) neutral pions by counting Dalitz pairs:

$$\frac{\Gamma_{2(000)}}{\Gamma_{2(L)}+\Gamma_{2(+-0)}}=0.38\pm0.07. \tag{27}$$

Together with our branching ratio for $(\pi^+\pi^-\pi^0)$ decays, this gives

$$\Gamma_{2(000)}/\Gamma_{2(+-0)}=2.42\pm0.63. \tag{28}$$

This differs from the prediction by 0.64 standard deviations.

Another prediction of the $\Delta I=\frac{1}{2}$ rule relates the pion energy spectra in the $K\to3\pi$ decays.[11,12] The energy

TABLE III. The values for K^+ branching ratios which were used to construct Tables I and IV. The K^+ lifetime was taken to be $(1.224\pm0.013)\times10^{-8}$ sec.

Decay rate	Rates ($\times10^6$ sec^{-1})	
$\Gamma_{+(\pi e\nu)}$	4.09 ± 0.41[a]	3.93 ± 0.36
$\Gamma_{+(\pi\mu\nu)}$	3.92 ± 0.49	3.30 ± 0.33[b]
$\Gamma_{+(++-)}$	4.66 ± 0.25	4.67 ± 0.14
$\Gamma^+{}_{(+00)}$	1.39 ± 0.16	1.36 ± 0.09

[a] Values obtained from the K^+ branching ratios of Roe *et al.* (Ref. 33).
[b] Values obtained by averaging the Xe bubble chamber and emulsion branching ratios. The K_{L3} rates of Birge *et al.* (Ref. 34) and Taylor *et al.* (Ref. 35) are not in very good agreement with those of Roe *et al.* and Alexander *et al.* (Ref. 34). If the former are excluded, the resulting averages are essentially equal to those given under Ref. a.

[32] F. S. Crawford, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 827.
[33] B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961).
[34] R. W. Birge, D. H. Perkins, J. E. Peterson, D. H. Stork, and M. W. Whitehead, Nuovo Cimento 4, 834 (1956); G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, Nuovo Cimento 6, 478 (1957).

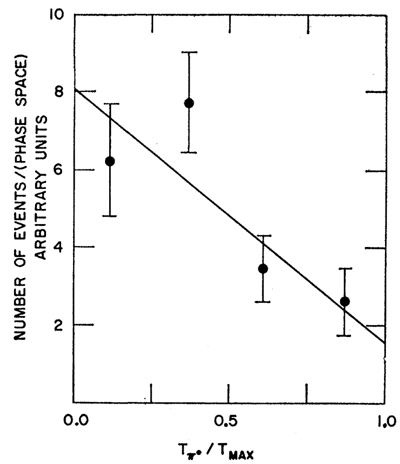


FIG. 8. The energy spectrum of the π^0 in $(\pi^+\pi^-\pi^0)$ decay, divided by the phase space distribution. The energy of the π^0 is given as a fraction of the maximum value.

spectrum of the π^0 in $(\pi^+\pi^-\pi^0)$ decay is shown in Fig. 8, as the ratio of the number of events in each of four regions to the integral of phase space over that region. The 83 events used are all the cases with $(P_0')^2 > -5000$ $(\mathrm{MeV}/c)^2$ with acceptable $P_{K_2^0}$. The analysis in Sec. VI indicated that ~ 77 of these are $(\pi^+\pi^-\pi^0)$ decays; the other six are $(\pi\mu\nu)$ decays. A least-squares fit to the π^0 spectrum with the form

$$W=\left(1+\frac{aT_{\pi^0}}{M_K}\right)\Phi, \tag{29}$$

where Φ is a phase space factor given by Weinberg,[12] gives

$$a_{(+-0)}=-7.3\pm1.6. \tag{30}$$

[By examining the artifically generated $(\pi\mu\nu)$ cases with $(P_0')^2\sim 0$, it was found that this result is not sensitive to the $(\pi\mu\nu)$ contamination. Also, this is indicated by the value of $a_{(+-0)}$ for those events with $(P_0')^2>0$, which is 8 ± 2.] The prediction of $\Delta I=\frac{1}{2}$ is that this should be the same spectrum as that observed in τ' decay, where Bjorklund *et al.*[35] have found

$$a_{\tau}'=-7.1\pm1.7. \tag{31}$$

The agreement with $\Delta I=\frac{1}{2}$ is very good, but this may just be a property of the symmetry of the pion states and the nature of the final-state interactions. For example, in $\eta\to\pi^+\pi^-\pi^0$ the three pions have isotopic spin one as a consequence of the assignment of quantum numbers $I=0$, $G=+1$ to the η.[36] Berley *et al.*[37] have shown that the π^0 spectrum in this decay is in agreement with the assumption that the matrix elements for τ' and for η are proportional. It seems probable then, that the variation in the τ' matrix elements is due to final state interactions rather than the structure of the weak interaction. But under this assumption, the three pion state will have either $I=1$ or $I=3$. Consequently, a $\Delta I=\frac{3}{2}$ interaction will only be able to connect the K (with $I=\frac{1}{2}$) with the $I=1$ state, which is also the state required by the $\Delta I=\frac{1}{2}$ rule. This result, then, may be sensitive only to $\Delta I=\frac{5}{2}$ interactions. A similar argument may be applied to the Eq. (25).

A test which is free from this objection can be made by combining our measurement of

$$\Gamma_{2(+-0)}/\Gamma_{2(L)}=0.186\pm0.036 \tag{32}$$

with a measurement of $\Gamma_{2(L)}$. Using the value of Alexander *et al.*[16]

$$\Gamma_{2(L)}=(9.31\pm2.49)\times10^6/\mathrm{sec}, \tag{33}$$

we obtain

$$\Gamma_{2(+-0)}=(1.74\pm0.57)\times10^6/\mathrm{sec}, \tag{34}$$

which may be compared with that given by (22) using the average value for $\Gamma_{+(+00)}$,

$$\Gamma_{2(+-0)}=(2.87\pm0.23)\times10^6/\mathrm{sec}, \tag{35}$$

or a discrepancy of 1.8 standard deviations. Another value of $\Gamma_{2(L)}$ may be obtained by combining our value of $\Gamma_{2(+-0)}/\Gamma_{2(L)}$ with the measurement of Darmon *et al.*[38] who find

$$\Gamma_{2(L)}+\Gamma_{2(+-0)}=(17.0\pm6.5)\times10^6/\mathrm{sec}. \tag{36}$$

This yields

$$\Gamma_{2(L)}=(13.8\pm5.3)\times10^6/\mathrm{sec}. \tag{37}$$

Finally, we may use our data and that of Anikina *et al.*[5] to transform the measurement of the K_2^0 lifetime by Bardon *et al.*[2] to a leptonic rate:

$$\Gamma_{2(L)}=(7.25_{-2.1}^{+3.1})\times10^6/\mathrm{sec}. \tag{38}$$

The weighted average of the three values of $\Gamma_{2(L)}$ is

$$\Gamma_{2(L)}=(9.9\pm2.0)\times10^6/\mathrm{sec}. \tag{39}$$

The value of $\Gamma_{2(L)}$ predicted by the $\Delta I=\frac{1}{2}$ current rule is, using the $\Gamma_{+(L)}$ of Roe *et al.*

$$\Gamma_{2(L)}=(16.0\pm0.9)\times10^6/\mathrm{sec} \tag{40}$$

or 2.6 standard deviations from the average experimental value.

These comparisons are summarized in Table IV, together with some others using averages of different combinations of the experimental values. In particular, when the average value of $\Gamma_{2(+-0)}/\Gamma_{2(L)}$ is combined with the average $\Gamma_{2(L)}$, the result is only 1.4 standard deviations from the $\Delta I=\frac{1}{2}$ value.

Our conclusion is that the $\Delta I=\frac{1}{2}$ rule gives a satisfactory account of the $K\to3\pi$ decays, within the precision of the present data.

IX. $K_2^0\to\pi^+\pi^-$

If both CP invariance and the $\Delta I=\frac{1}{2}$ rule are violated, the K_2^0 can decay into two charged pions.[22] Also, it has

[35] S. Bjorklund, E. L. Koller, and S. Taylor, Phys. Rev. Letters 4, 424 (1960); 4, 475 (1960) (*E*).

[36] G. Feinberg and A. Pais, Phys. Rev. Letters 9, 45 (1962).

[37] D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters 10, 114 (1963).

[38] J. Darmon, A. Rousset, and J. Six, Phys. Letters 3, 57 (1962).

TABLE IV. A summary of experimental results and the prediction of the $\Delta I=\frac{1}{2}$ rule and $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ currents applied separately.

Quantity	Experimental Value	$\Delta I=\frac{1}{2}$ Rule	$\Delta I=\frac{1}{2}$ Currents	$\Delta I=\frac{3}{2}$ Currents
$\dfrac{\Gamma_{2(+-0)}}{\Gamma_{2(L)}+\Gamma_{2(+-0)}}$	0.157 ± 0.03 [a] $0.185_{-0.034}^{+0.038}$ [b] 0.171 ± 0.023 [c]			
$\dfrac{\Gamma_{2(000)}}{\Gamma_{2(L)}+\Gamma_{2(+-0)}}$	0.38 ± 0.07 [d]			
$\dfrac{\Gamma_{2(000)}}{\Gamma_{2(+-0)}}$	2.42 ± 0.63 [e] 2.22 ± 0.50 [f]	2.00 ± 0.19 [n] 1.84 [o]		
$\Gamma_{2(L)}$	$9.31\pm2.49\times(10^6\ \mathrm{sec}^{-1})$ [g] 13.8 ± 5.3 [h] $7.25_{-2.1}^{+3.1}$ [i] 9.9 ± 2.0 [j]		16.0 ± 1.3 [q] 14.5 ± 1.0 [r]	4.00 ± 0.31 [q] 3.62 ± 0.25 [r]
$\Gamma_{2(+-0)}$	1.74 ± 0.57 [k] 1.84 ± 0.52 [l] 2.04 ± 0.50 [m]	2.81 ± 0.19 [p]		

[a] Present experiment.
[b] Astier *et al.* (Ref. 18).
[c] Average of Ref. a and Ref. b.
[d] Anikina *et al.* (Ref. 5).
[e] From present experiment and Ref. d.
[f] From Ref. c and Ref. d.
[g] Alexander *et al.* (Ref. 16).
[h] From Darmon *et al.* combined with Ref. a (Ref. 38).
[i] From Bardon *et al.* combined with Ref. a (Ref. 2).
[j] Weighted average of Ref. g, Ref. h, and Ref. i. Since the errors are essentially statistical, the errors of each value were divided by a $(\Gamma_{2(L)})^{1/2}$ in determining the weighted average, to give unbiased weights.
[k] From the present experiment and Ref. g.
[l] From the present experiment and Ref. j.
[m] From Ref. c and Ref. j.
[n] From Eq. (25) and the average K^+ rates, Table III, Ref. a.
[o] From the $\Delta I=\frac{1}{2}$ rule, assuming only symmetrical final states.
[p] From Eq. (22) and Table III, Ref. a.
[q] Using Table III, Ref. a.
[r] Using Table III, Ref. b.

been demonstrated that the absorption of K_2^0's in matter causes a coherent regeneration of a K_1^0 component which then decays into two pions.[4] Both of these processes would give rise to apparent K_1^0's which point directly at the target which is the source of the K_2^0 beam.

The best upper limit for the $K_2^0\rightarrow\pi^+\pi^-$ decay is $\sim1\%$.[2,23] The number of K_1^0 coherent regenerations in the hydrogen (due to the nuclear interaction) may be estimated from the formula of Good,[39] and is less than one event in the present run, because of the low density of liquid hydrogen.

We have searched for such events by examining the distribution in $M_{\pi^+\pi^-}$ and $\cos\theta_T$ for all the V^0's without identified leptons, where $M_{\pi^+\pi^-}$ is the mass of the particle decaying into π^+ and π^-, and θ_T is the angle between the momentum vector of the V^0 and the line from the vertex of the V^0 to the center of the target. There is no significant evidence for a peak at $M_{\pi^+\pi^-}=M_{K^0}$ and $\cos\theta_T=1$, above the smoothly varying background of ($\pi e\nu$) decays with small ν momentum. An *upper limit* for the rate of two pion decays can be obtained by taking all events with $M_{\pi^+\pi^-}=M_{K^0}\pm20$ MeV and $0.997\leqslant\cos\theta\leqslant1.000$. There are nine events in this region. Since the fiducial volume in the chamber is more than two K_1^0 decay lengths from the wall of the bubble chamber, no correction is necessary for K_1^0 mesons regenerated in the wall. Also, the number of incoherently regenerated K_1^0's from hydrogen in this angular interval, where the recoil protons would be too short to be seen, is less than one-tenth event. Consequently, the branching ratio for $K_2^0\rightarrow\pi^+\pi^-$ may be obtained from the nine events, after applying the scanning efficiency and adding to the number of K_2^0 decays seen the expected number of $K_2^0\rightarrow\pi^0\pi^0\pi^0$:

$$\frac{K_2^0\rightarrow\pi^+\pi^-}{all\ K_2^0}<1.5\%. \qquad (41)$$

[39] M. L. Good, Phys. Rev. **106**, 591 (1957); **110**, 550 (1958).

X. CONCLUSIONS

In agreement with previous experiments, our results support the assumption of CP invariance in K_2^0 decays, on the basis of two tests. First, we place an upper limit of 1.5% on the fraction of K_2^0 decay into two charged pions, a state with CP eigenvalue opposite to that of the pure K_2^0. Second, the charge ratio $(\pi^+e^-\nu)/(\pi^-e^+\nu)$ is equal to unity, within the statistical error. This result would follow from CP invariance, but an argument of Weinberg[22] shows that the limit on two pion decay places limits on this ratio: $0.92<(\pi^+e^-\nu)/(\pi^-e^+\nu)<1.08$. It must be noted that these results would follow even if CP were not conserved, if there were no $\Delta I=\frac{3}{2}$ amplitude in K^0 decay to two pions. At the time these arguments were originally made, it seemed that the $K_1^0 \to \pi^0\pi^0/K_1^0 \to \pi^+\pi^-$ ratio required some contribution $\Delta I=\frac{3}{2}$, but with the recent data, this is no longer true.[32]

By the analysis of the energy correlations in $(\pi e\nu)$ decay we have established that, if the interaction is of a single type, it is almost certainly vector. The scalar interaction is possible only if there is an unusual and specific variation of the form factor, and tensor is definitely excluded.

The form factor for the vector interaction is consistent with a constant value, or a small decrease, in accordance with calculations taking into account $K\pi$ resonances and the possible weakly interacting vector boson.

The correlation data for the $(\pi e\nu)$ decays are not a perfect fit to the vector decay interaction, although the disagreement is not statistically significant. It has been pointed out that such an effect could be due to a nonlocality in the $e\nu$ interaction.[40] Further investigation of this point may well be worthwhile. It has been pointed out in the discussion of the branching ratios that our value of the $(\pi\mu\nu)/(\pi e\nu)$ ratio depends on the assumption of locality.

The branching ratios may be considered to determine two independent quantities: the fraction of the charged decays giving three pions; and the ratio $(\pi\mu\nu)/(\pi e\nu)$. The former agrees very well, perhaps fortuitously, with the value calculated by assuming the validity of the $\Delta I=\frac{1}{2}$ rule and $\Delta I=\frac{1}{2}$ currents. Combining this result with absolute decay rate measurements allows a test of the $\Delta I=\frac{1}{2}$ rule alone: the experimental result is in fair agreement with the prediction.

The ratio $(K_2^0 \to \pi^0\pi^0\pi^0)/(K_2^0 \to \pi^+\pi^-\pi^0)$, which we have obtained by combining our data with that of Anikina *et al.*, is in good agreement with the $\Delta I=\frac{1}{2}$ rule. The π^0 spectrum in the $(\pi^+\pi^-\pi^0)$ decays is also in good agreement with the $\Delta I=\frac{1}{2}$ rule, but these two results are not tests of the $\Delta I=\frac{1}{2}$ rule if symmetrical pion states dominate and the pion distribution is a result of final state interactions. This point is emphasized by the similarity of the π^0 spectrum in $(\pi^+\pi^-\pi^0)$ decay and in the decay $\eta \to \pi^+\pi^-\pi^0$.

The $(\pi\mu\nu)/(\pi e\nu)$ ratio is 0.73 ± 0.15. The value in K^+ decay is somewhat larger than, but statistically consistent with this value. The prediction of the partially conserved strange current theory is ~0.68, which would agree with our value. If $\Delta I=\frac{3}{2}$ currents exist, this ratio could be different for K_2^0 and K^+ decay. [*Note added in proof*. Further measurements of the K^+ branching ratios by Roe *et al.* give $R=0.65\pm0.2$. in good agreement with the value for K_2^0 in this experiment. The agreement with the $\Delta I=\frac{1}{2}$ currents is also improved.]

XI. DISCUSSION

All the results of this experiment alone support what one might consider the "conventional" theory of weak interactions: the $K_2^0 \to \pi e\nu$ decay is caused by the vector interaction with an approximately constant form factor, and conservation of CP invariance; the branching ratio for $(\pi^+\pi^-\pi^0)$ decay is given by assuming the $\Delta I=\frac{1}{2}$ rule holds and that only $\Delta I=\frac{1}{2}$ currents are present; and the ratio of $(\pi\mu\nu)/(\pi e\nu)$ decay rates is only somewhat greater than the ratio of phase space volumes, as predicted, for example, by the theory of the partially conserved vector current. This agreement is somewhat surprising, since several recent experiments on other aspects of strange particle decay have contradicted the above picture, without disagreeing with the specific results of this experiment.

Foremost among these contradictions is that provided by the experiment of Ely *et al.*[15] on K^0 decay, where the transition $K^0 \to \pi^+ + e^- + \nu^-$ is observed, which proceeds only by the interaction of $\Delta I=\frac{3}{2}$ currents. This conclusion has been supported by the observation of the decay $\Sigma^+ \to \eta + \mu^+ + \nu$.[17] If such currents exist, they may contribute to K_2^0 decay, and, since the nature of this interaction is unknown, it is no longer clear that the $(\pi e\nu)$ decay should be expected to proceed by a dominately vector interaction.

When the branching ratios measured in this experiment are combined with a measurement of the absolute decay rate of the K_2^0, it is found that the rate for $(\pi^+\pi^-\pi^0)$ decay is in fair agreement with the $\Delta I=\frac{1}{2}$ rule, although the rate for K_2^0 leptonic decays is considerably smaller than that given by the $\Delta I=\frac{1}{2}$ current prediction. That this is possible despite the agreement with $\Delta I=\frac{1}{2}$ currents in Table I, is due to the relatively large errors involved; if the rates are equal to the measured values, a $\Delta I=\frac{3}{2}$ amplitude equal to $\sim11\%$ of the $\Delta I=\frac{1}{2}$ amplitude in three pion decay would be required.[41]

Currents with $\Delta I=\frac{3}{2}$ would allow a different $R=(\pi\mu\nu)/(\pi e\nu)$ ratio for K^+ and K_2^0, but, within the accuracy of the present experiments, there is no such effect. The parameter ξ which is related to R by Eq. (20), can also be determined from the energy correlations in $(\pi\mu\nu)$ decay. It was not possible to study these correlations in this experiment, but they have been studied in two K^+ experiments, with results which are contradictory, at least if a constant $(\pi e\nu)$ form factor is assumed. Attempts have been made to find a model which is consistent with both experiments, and the measured value of R for K^+. This was found to be possible, if the $(\pi e\nu)$ form factor in K^+ decay has a strong energy dependence, *vanishing* for a pion momentum of ~85 MeV/c.[42] This implies a value of R appreciably greater than 0.65. The $(\pi e\nu)$ form factor we measure in K_2^0 decay does *not* seem to have this property; consequently, *if* this picture of the K^+ decay is confirmed, $\Delta I=\frac{3}{2}$ currents must be very important.

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[40] C. Chahine and B. Jouvet (unpublished).

[41] R. H. Dalitz, Rev. Mod. Phys. **31**, 823 (1959).

[42] G. Zweig, Phys. Rev. **130**, 2449 (1963); N. Brene, D. Egardt, B. Quist, and D. A. Geffen, Nucl. Phys. **30**, 399 (1962).

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APPENDIX

An estimate may be made of the number of $K_2{}^0$'s incorrectly identified as examples of $(\pi e\nu)$ decay. A mu meson of 200 MeV/c has a bubble density 1.30 times minimum, and a pi meson of that momentum is 1.49$\times$ minimum. Since the average standard deviation of a bubble density measurement was $\sim$20% (from statistics and reference-track variation), about 22% of 200-MeV/c muons would be expected to be identified as electrons, and vice versa. Only 4% of the 200-MeV/c pions would be confused; however, there are about twice as many pions as muons in the sample of decay tracks. These percentages decrease rapidly with momentum: few tracks with momentum less than 150 MeV/c will be incorrectly identified.

The number of pions and muons with momentum around 200 MeV/c can be obtained from the momentum distribution of the decay tracks not identified as electrons, and the fraction of these which are muons from the branching ratios. Then, using the probabilities of misidentification it was found that $\sim$7.5 electrons would be expected to be lost, and that $\sim$5.5 muons and $\sim$1.5 pions would be identified as electrons. Therefore, the total number of events is approximately correct but $\sim$7 out of the 153 have incorrect configurations.

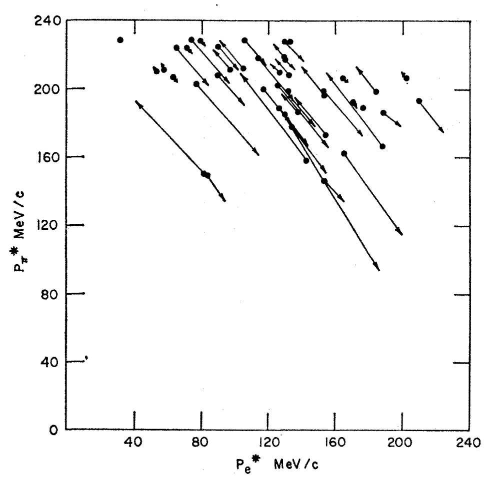

FIG. 9. This plot illustrates the effect of the ambiguity in $P_{K_2{}^0}$ on the momenta of the pion and electron in the $K_2{}^0$ rest frame, for a sample of events. The points are the configurations selected, while the head of the arrow indicates the other possible configuration.

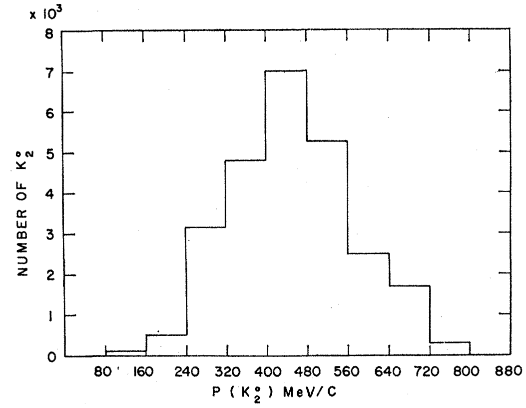

FIG. 10. The $K_2{}^0$ momentum distribution.

An event identified as $(\pi e\nu)$ decay still has two possible $K_2{}^0$ momenta, and therefore two different configurations in the center of mass. In practice, this problem is usually not serious. A large portion, 57%, of the events give an impossible value for one of the momenta; for example, a negative value, or one greater than that kinematically possible. Others, 29%, give two values sufficiently near in value that the center-of-mass values of T_e and T_π are equal, within 20 MeV. For the remaining 14%, the $K_2{}^0$ momentum was chosen which was nearer to one of the two nominal $K_2{}^0$ momenta which would have resulted from the use of a hydrogen target. At least half of these assignments should be correct, leaving $\sim$7% of the cases with incorrect transformations to the center of mass. The values of T_e and T_π would change on the average by about 35 MeV and 25 MeV, respectively, for such a change in $P_{K_2{}^0}$, as shown in Fig. 9.

The distribution of $K_2{}^0$ momenta obtained in this fashion from the $(\pi e\nu)$ events is shown in Fig. 10. In this figure, the distribution is given in terms of the integrated flux per momentum interval. This is done by using the $(\pi e\nu)$ branching ratio and a $K_2{}^0$ lifetime from the $\Delta I=\frac{1}{2}$ rules.